

Mathathon · Algebra 2

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May 16, 2024

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1 Introduction

What is Algebra?

“Algebra is the branch of mathematics that studies algebraic structures and the manipulation of statements within those structures (“Algebra,” 2024).”

This can mean anything from adding or subtracting a variable from both sides of an equal sign to more advanced branches such as abstract algebra and linear algebra.

What is Algebra 2?

Algebra 2 is part of the elementary branch of algebra and is a more advanced version of algebra 1.

That is, Algebra 2 mainly consists of equations, variables, and other equations and how to manipulate them, with a large focus on polynomials.

2 Beginner Information

What is a polynomial?

A polynomial is an expression made up of variables and constants. An example of a polynomial would be $2x^2 + 5$. The aforementioned polynomial is an example of a binomial because it has 2 terms. It also has a degree of 2, because the highest exponent of x is 2.

What is a logarithm?

A logarithm is a way to write exponential expressions. For example, $3^6 = 729$ could be written as $\log_3(729) = 6$. This can be generalized to the following.

$$a^b = c \equiv \log_a(c) = b$$

Because of this, we can use logarithms to solve for unknown exponents.

Example

Solve for x in $5^x = 625$.

Solution

$$\begin{aligned}5^x &= 625 \\ \log_5(625) &= x \\ \log_5(625) &= 4 \\ \therefore x &= 4\end{aligned}$$

What are radians and degrees?

Radians and degrees are both units used to measure angles. Degrees are denoted with the $^\circ$ or deg symbols. Radians are denoted with the rad symbol.

The angle measure of a full circle is equal to 2π rad or 360 deg.

You can convert degrees to radians by multiplying the degrees by $\frac{\pi}{180}$ and radians to degrees by multiplying the radians by $\frac{180}{\pi}$.

3 Beginner Problems

What is the degree of the following polynomial?

$$3x^4 - 2x^2 + x - 2$$

Solve for x in the following equation.

$$\frac{2x - 4}{5x + 2} = 2$$

Rewrite $4^3 = 64$ in logarithmic form.

Convert 270° to radians.

Let $f(x) = x^x + x$. Find $f(3)$.

4 Intermediate Information

What are negative exponents?

Regular exponents represent repeated multiplication. For example, $9^4 = 9 \cdot 9 \cdot 9 \cdot 9$. This can be generalized to the following.

$$a^n = \underbrace{a \cdot a \cdot a}_{n \text{ times}}$$

Negative exponents simply go in the other direction. That is, you repeatedly divide instead of repeatedly multiplying. For example, $2^{-3} = \frac{\frac{1}{2}}{2}$. This can be rewritten as $\frac{1}{2^3}$. Negative exponents can be generalized as the following.

$$a^{-n} = \frac{\frac{1}{a}}{a} \left. \vphantom{\frac{1}{a}} \right\} n \text{ times} = \frac{1}{a^n} = \frac{1}{\underbrace{a \cdot a \cdot a}_{n \text{ times}}}$$

What are translations?

“[A] translation is a geometric transformation that moves every point of a figure, shape or space by the same distance in a given direction (“Translation (geometry),” 2024).”

In other words, translations are the moving of shapes and equations. To shift $f(x)$ a units right and b up, we can change $f(x)$ to $f(x - a) + b$.

Example

Translate $f(x) = 2x^2$ 2 units left and 1 unit down. Let this translated function be $g(x)$.

Solution

$$f(x) = 2x^2$$

$$f(x + 2) - 1 = 2(x + 2)^2 - 1$$

$$g(x) = 2(x + 2)^2 - 1$$

$$g(x) = 2(x^2 + 4x + 4) - 1$$

$$g(x) = 2x^2 + 8x + 7$$

5 Intermediate Problems

Let $f(x) = x^x + x$. Find $f(-2)$.

What does x equal in the following equation?

$$x = \log_2(8)$$

Let $f(x) = x^3 - 2x$. Let $g(x)$ be $f(x)$ translated 2 units up and 3 units right. Find $g(x)$.

6 Advanced Information

What are systems of equations?

Systems of equations are sets of 2 or more equations. To solve a linear system of equations, you find the point(s) where the equations intersect.

Example

Solve the following linear system of equations.

$$\begin{cases} 6y - 2 = x \\ y = 3x - 2 \end{cases}$$

Solution

$$\begin{cases} 6y - 2 = x \\ y = 3x - 2 \end{cases}$$

$$\begin{array}{ll} 6y - 2 = x & y = 3x - 2 \\ 6(3x - 2) - 2 = x & y = 3(6y - 2) - 2 \\ 18x - 12 - 2 = x & y = 18y - 6 - 2 \\ 18x - 14 = x & y = 18y - 8 \\ 18x = x + 14 & y - 18y = -8 \\ 18x - x = 14 & -17y = -8 \\ 17x = 14 & y = \frac{-8}{-17} \\ x = \frac{14}{17} & y = \frac{8}{17} \end{array}$$

$$\begin{cases} 6y - 2 = x \\ y = 3x - 2 \end{cases} = \left(\frac{14}{17}, \frac{8}{17} \right)$$

How do you find the average value of a linear function?

You can find the average value of a linear function f on the interval $[a, b]$ by taking the average of f at a and f at b . The average value of a linear function f on the interval $[a, b]$ is can be written as the following.

$$f_{\text{avg}} = \frac{f(a) + f(b)}{2}$$

While it *is* possible to find the average value of a curve, doing so is beyond the scope of algebra 2.

7 Advanced Problems

Find the solution(s) to the following system of equations.

$$\begin{cases} y = 2x^2 - 5x + 4 \\ y = 3x - 4 \end{cases}$$



Let $f(x) = 2x$. What is the average value of the function on the interval $[0, 5]$?

Prove that $\sin^2(\theta) + \cos^2(\theta) = 1$.







8 References

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