Geometry/Trigonometry

With Hitej

What’s Geometry??

~ the branch of mathematics concerned with the properties and relations of points, lines, surfaces, solids, and higher dimensional analogs

In other words..

SHAPES

Transformations (oooooh ahhhhh)

The shape is.. changing?

Close, but by transformation, the shape generally remains the same size and, well, shape.

The 3 types of transformations that maintain this size and shape are translations, rotations, and reflections. These are called congruent transformations.

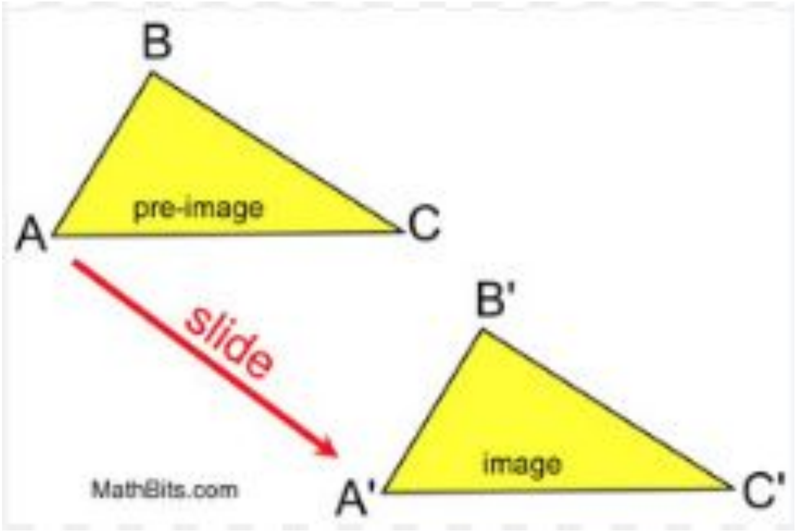
The last type that maintains the shape, but not the size is a dilation. It maintains the ratio of size, but not the size in general. The result is a shape that is “similar” to the input shape.

Translation

Translation

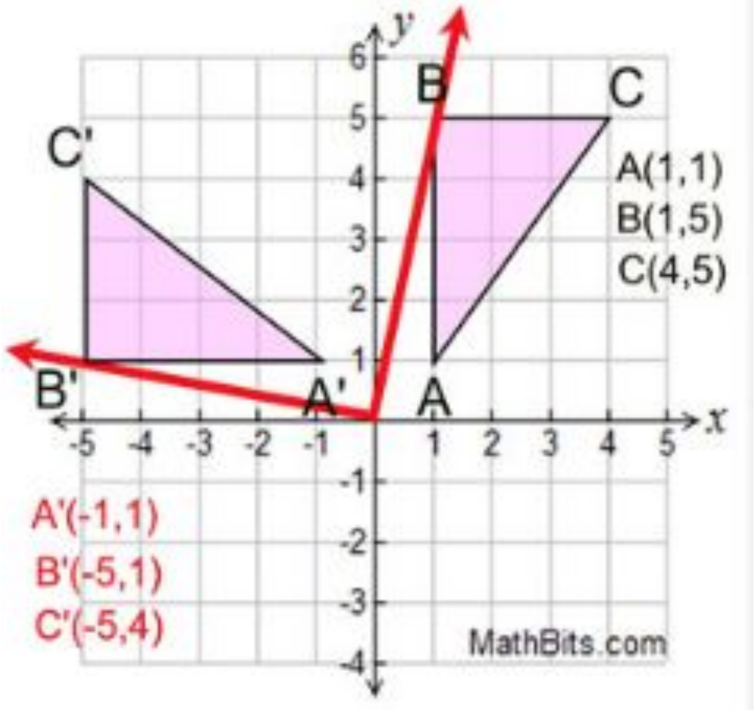
A translation can mean that the shape or object, or point or whatever is simply moved up or down or side to side. Think of a translation as just a movement.

It is denoted by that small apostrophe on the resulting figure shown.



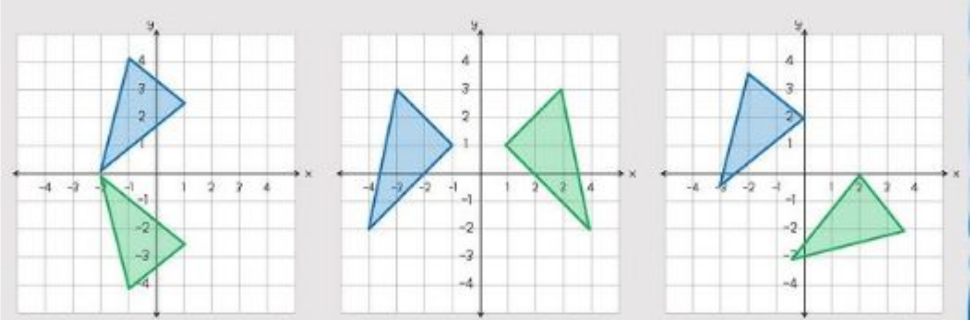
Rotation Rotation

As you can see, rotation is simply when the figure is rotated around a point.



Reflection noitcelfeR

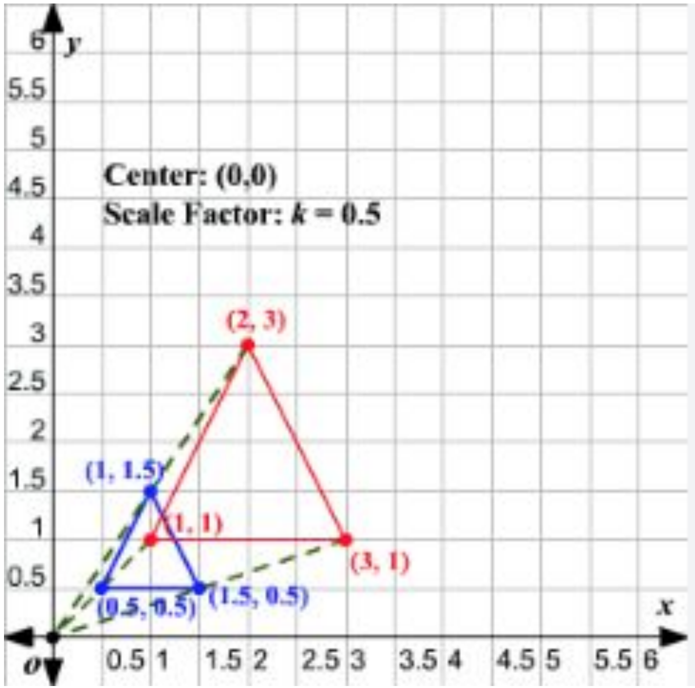
Think of reflection as a figure or shape looking at itself in the mirror. Now that mirror could be on the x, y, or any axis at all. It essentially just flips the shape over it like pictured on the right.



For all of these transformations, the shape has remained the same size.

Dilation! Dilation! Dilation!

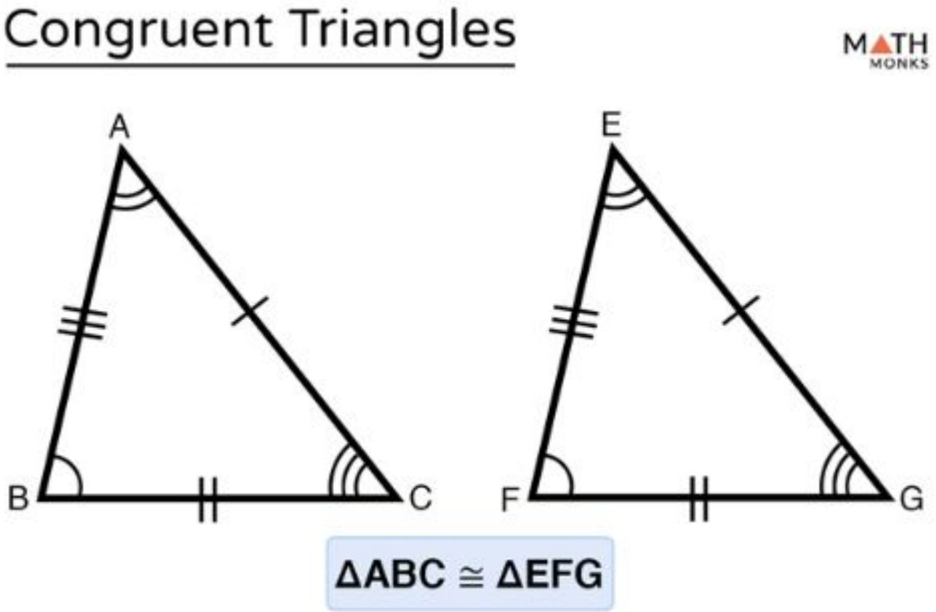
Dilation simply makes something bigger or smaller.



Congruence & Similarity

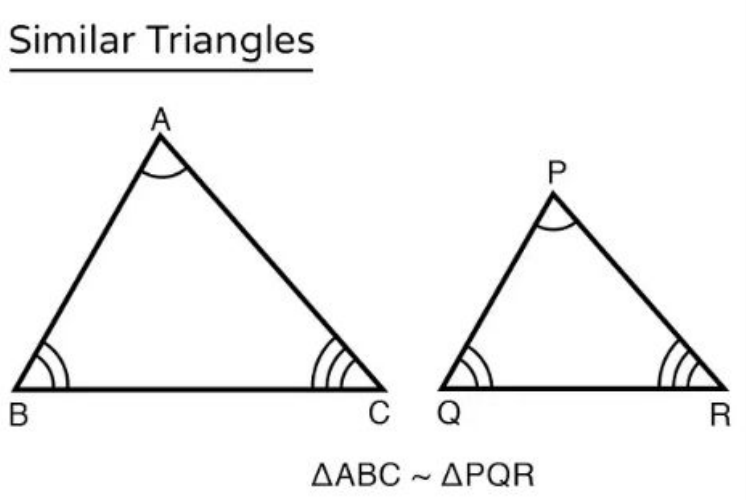
Congruence

Congruence is when corresponding sides and corresponding angles of a figure are equal.



Similarity

Corresponding angles are equal. From this, we know that the side lengths are scaled up or down, or even equal. Just because a triangle is similar doesn’t mean it can’t be congruent!



Can you guess what’s next?

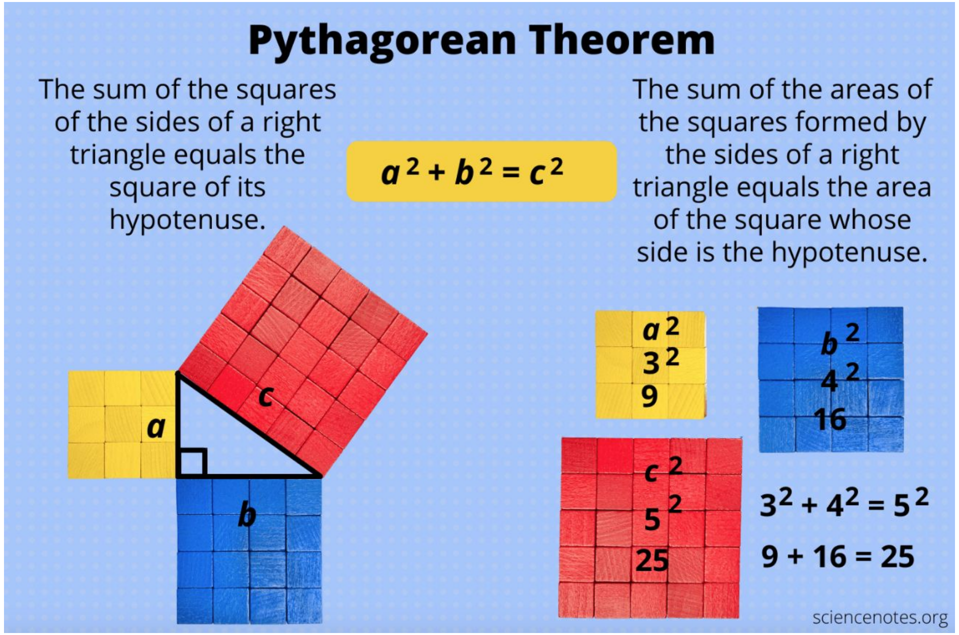
You've prob already learned it…

GOOD OL’ PYTHAGORAS

Pythagorean Theorem :0

The Pythagorean Theorem is AWESOME!

Also triangles :p



Distance Formula

How far am I from ice cream?

Well, that’s for me to know and you to find out.
Let’s say I am at (3, 4) and the ice cream is at (6, 12).

Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint Formula

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

How far am I from ice cream?

Well, that's for me to know and you to find out. Let's say I am at (3, 4) and the ice cream is at (6, 12).

6 - 3 is 3 and 12 - 4 is 8. 3^2 = 9 and 8^2 = 64.

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Or approximately 8.544.

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What’s Trigonometry??

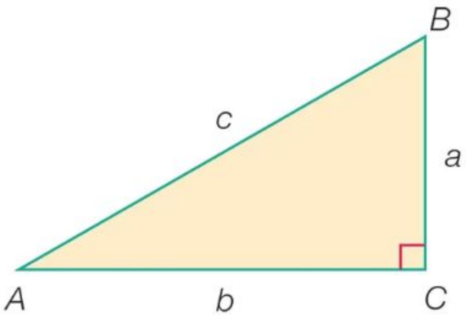
~ a branch of mathematics concerned with relationships between angles and side lengths of triangles

SOHCAHTOA

But… what does it mean??

SOH

SOH refers to the function called sine. OH refers opposite over hypotenuse. When entering the opposite side (opposite meaning opposite from an angle) divided by the hypotenuse side of a triangle we are given the angle it is relative to.



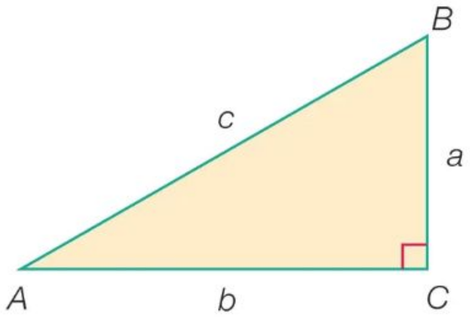
$$\sin A = \frac{a}{c} = \frac{\text{side opposite}}{\text{hypotenuse}}$$

$$\cos A = \frac{b}{c} = \frac{\text{side adjacent}}{\text{hypotenuse}}$$

$$\tan A = \frac{a}{b} = \frac{\text{side opposite}}{\text{side adjacent}}$$

CAH

CAH refers to the function called cosine. AH refers adjacent over hypotenuse. When entering the adjacent side (adjacent meaning next to an angle) divided by the hypotenuse side of a triangle we are given the angle it is relative to.



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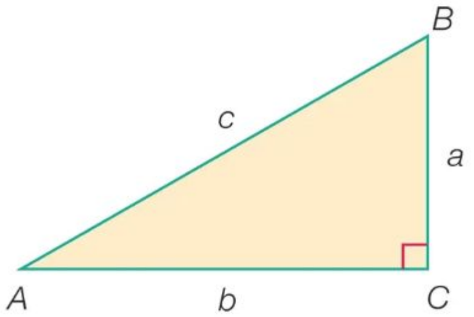
$$\tan A = \frac{a}{b} = \frac{\text{side opposite}}{\text{side adjacent}}$$

TOA

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

TOA refers to the function called tangent. OA refers opposite over adjacent. When entering the opposite side (opposite meaning opposite from an angle) divided by the adjacent side (adjacent meaning next to an angle) of a triangle we are given the angle it is relative to.



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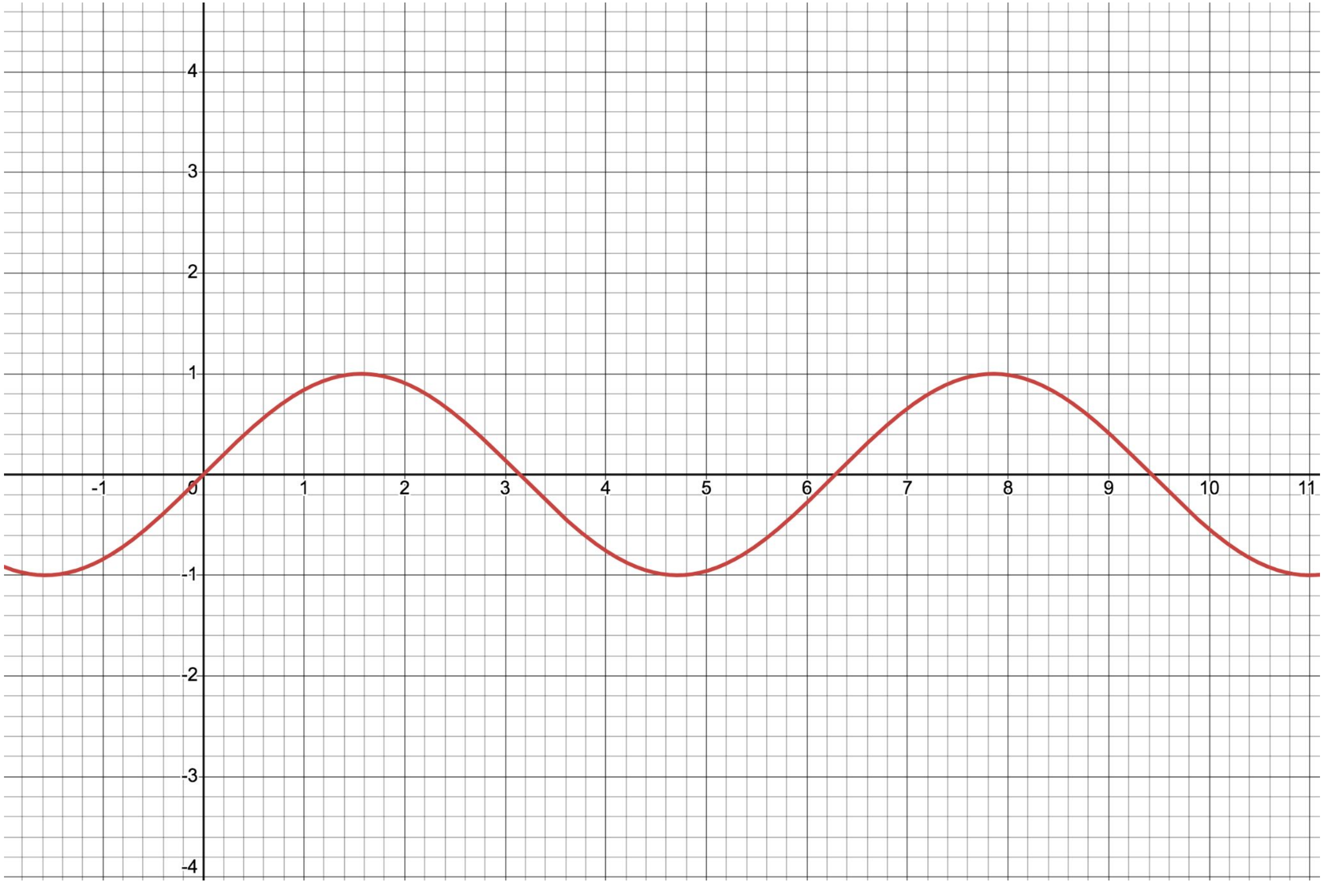
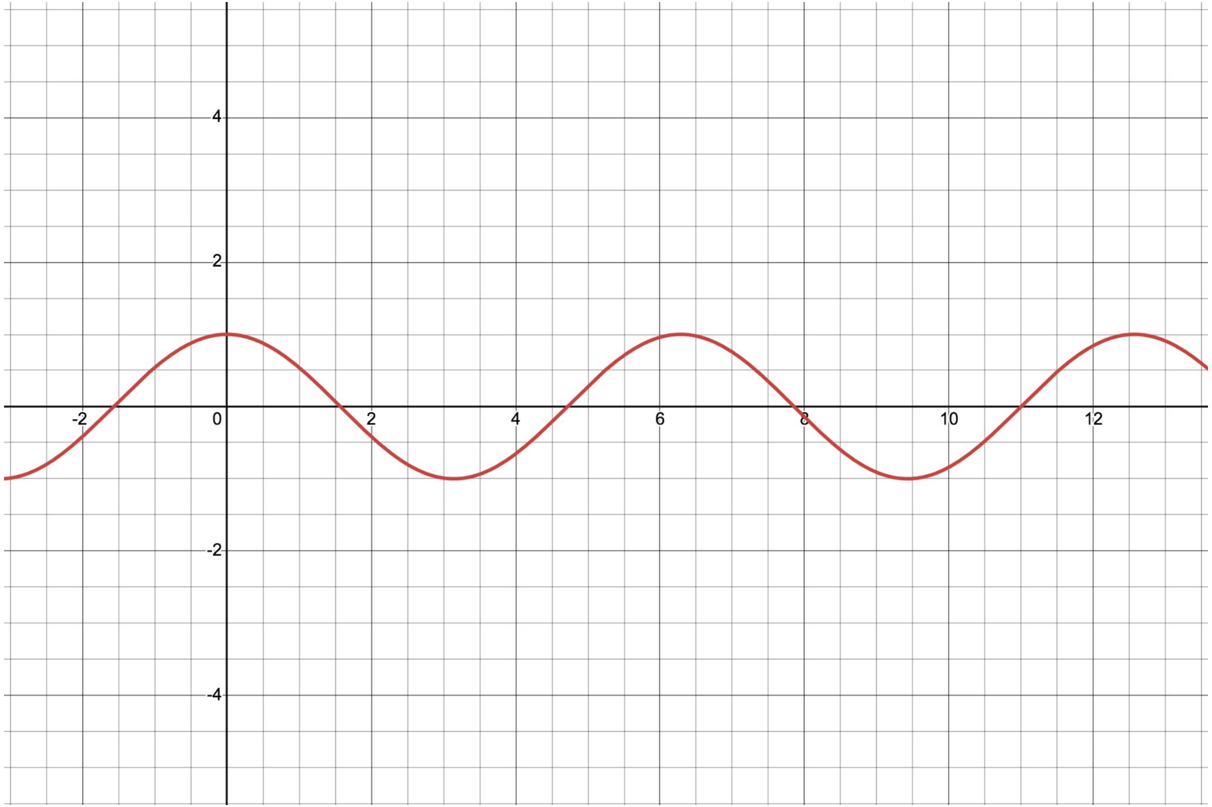
$$\tan A = \frac{a}{b} = \frac{\text{side opposite}}{\text{side adjacent}}$$

What do their graphs look like?

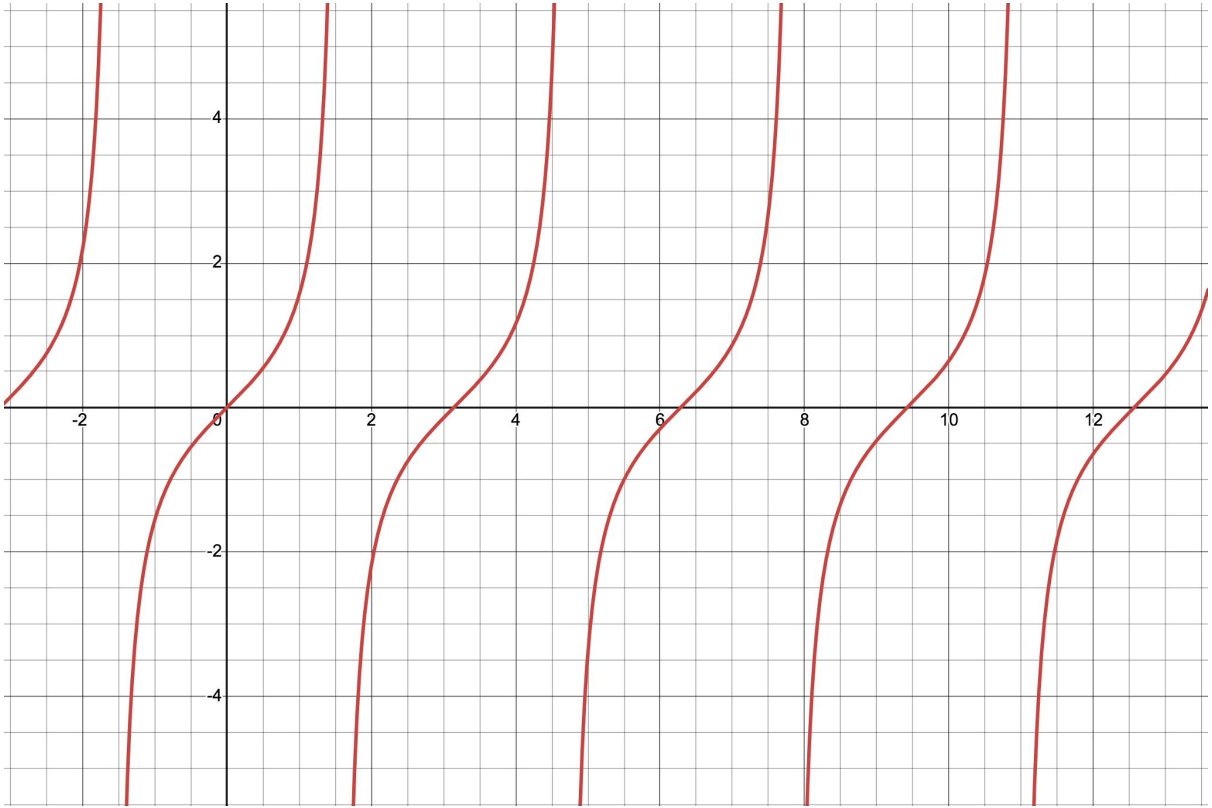
Sine



Cosine



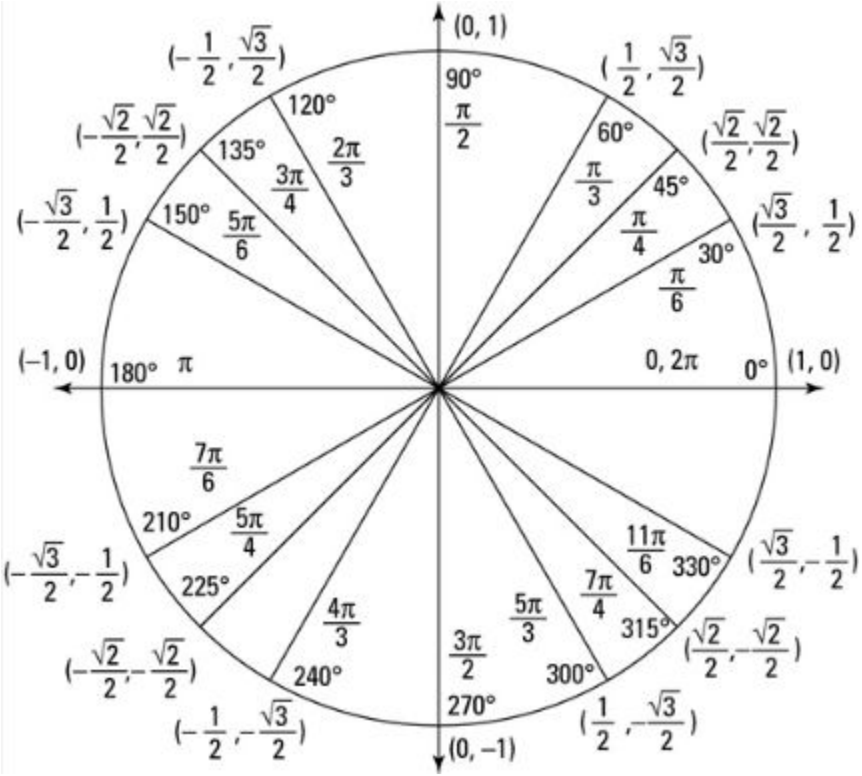
Tangent



Unit Circle :o

The unit circle is one of the most daunting topics in trigonometry, solely because of the whole jumble of numbers inside one small circle.

To properly understand this, I will first cover degrees, radians, and then jump right back in to how it relates to this crazy circle here.

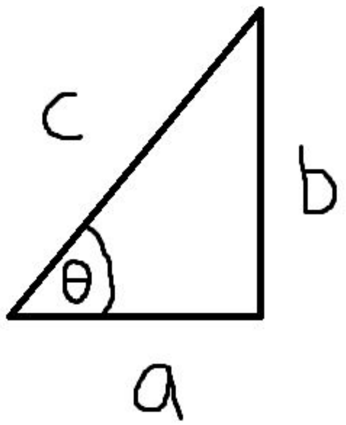


Degrees

I'm sure you all know degrees.

Have you ever heard that 360 degrees is a circle? Well, that value simply refers to the angle between two lines.

As you can see, this angle is generally denoted as the greek letter theta.

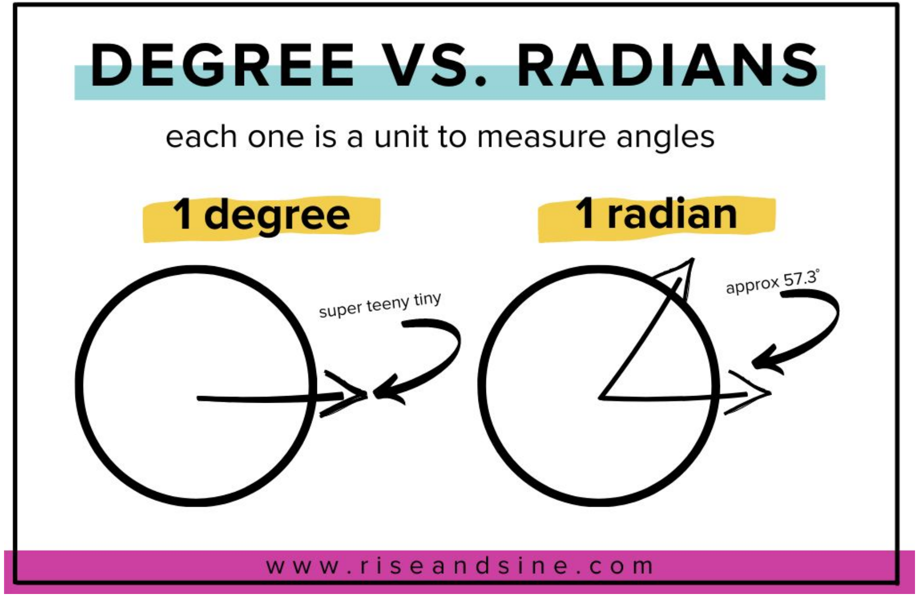


Radians

Radians is simply another way of measuring the angle theta.

360 degrees is equal to 2pi radians. This means a full circle is 2pi, a half circle is pi, and a quarter circle is pi/2.

This will make the unit circle a lot easier to understand.



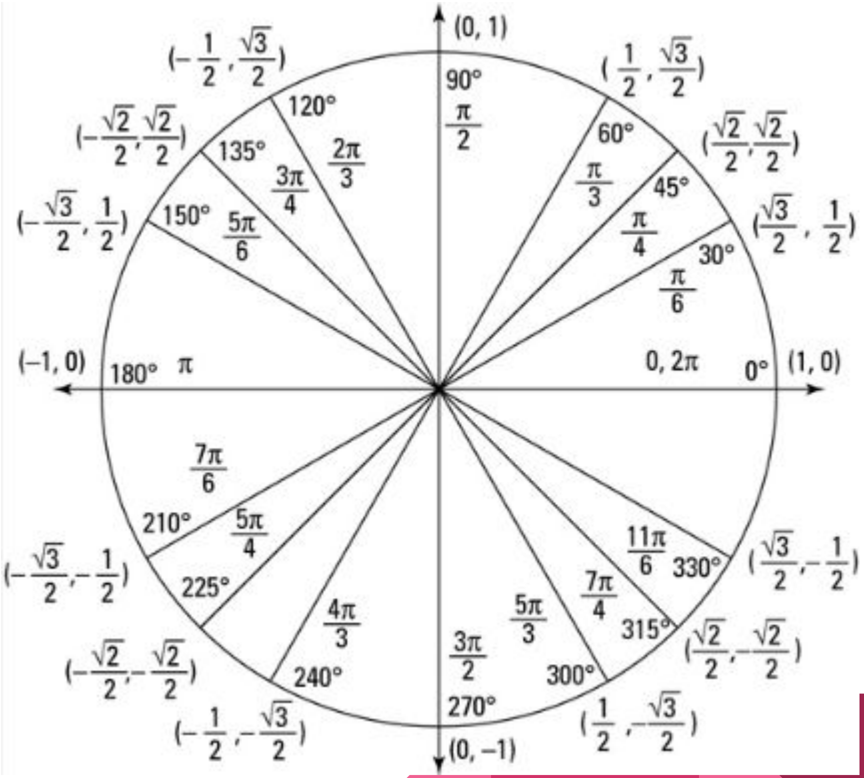
Unit Circle. Again.

I might need another slide..

Basically, what's happening here is that this is a circle with radius 1. The (x,y) coordinates around the circle correspond with the cos and sin values at these angles.

$(x, y) \rightarrow (\cos(\theta), \sin(\theta))$

Next slide..



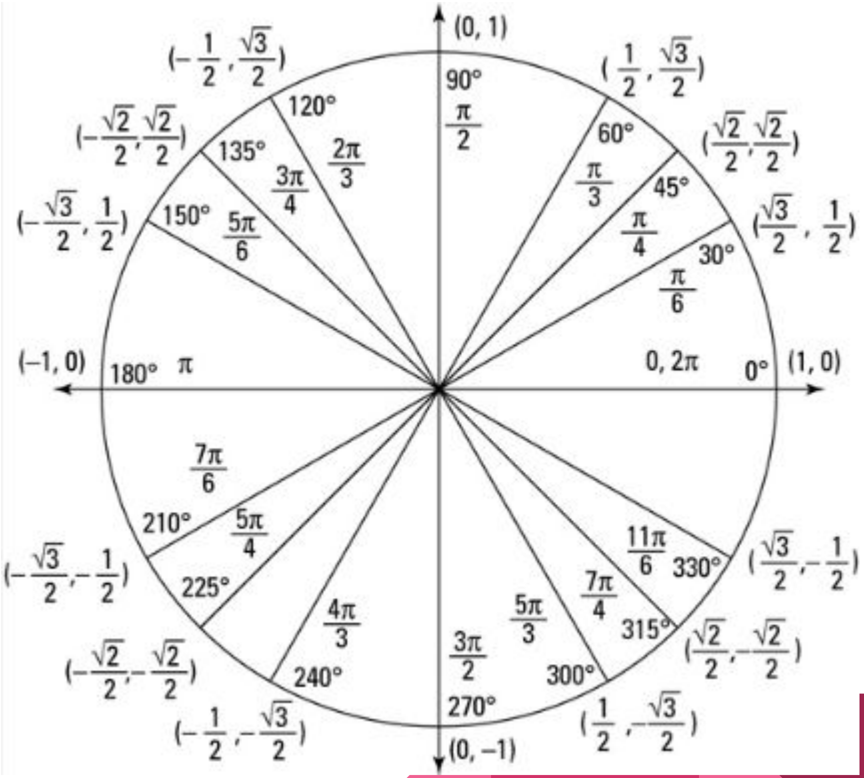
Unit Circle. Again. Again.

(x, y) → (cos(θ), sin(θ))

So, we can see at θ = 0 we have (1, 0). So we know that cos(0) and sin(0) are 1 and 0 respectively.

The unit circle is a handy tool that allows us to see these values at many common angles, and it’s given in both degrees and radians as you can see on the circle itself.

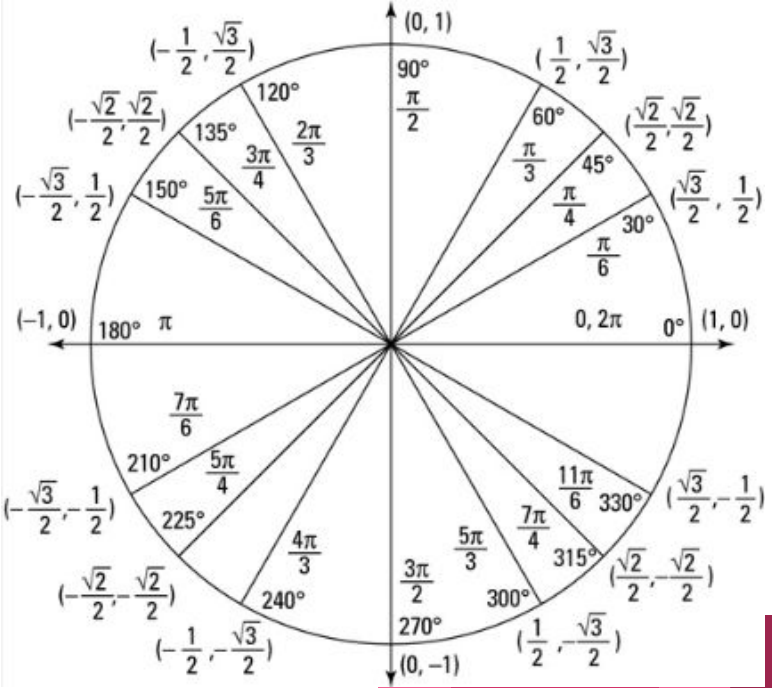
Next slide..



Unit Circle. Again. Again. Again.

$(x, y) \rightarrow (\cos(\theta), \sin(\theta))$

We also can use the unit circle and a trigonometric identity to find whether these functions output positives or negatives in each quadrant.



Next slide :/

PROOF: sin/cos = tan

$$\sin = \frac{O}{H}, \quad \cos = \frac{A}{H}$$

$$\frac{\left(\frac{O}{H}\right)}{\left(\frac{A}{H}\right)} = \frac{OH}{AH} = \frac{O}{A} = \underline{\underline{\tan}}$$

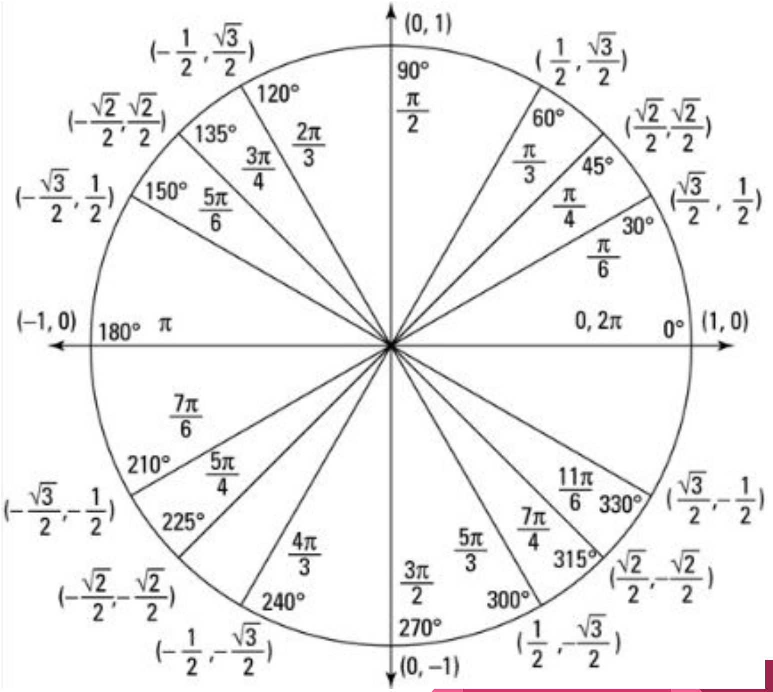
Unit Circle. Again. x4

Ok, so we know that sin is 0 when θ is 0 and cos is 1 when θ is 0.

If we look around at the x and y values on the unit circle, we see that everything is positive in the first quadrant, x is negative in the second quadrant, everything is negative in the third quadrant and y is negative in the fourth quadrant.

So sin is always positive in the second quadrant and cos is always positive in the fourth quadrant.

But what's positive in the third quadrant?



Unit Circle. Again. x5

Nothing is positive?

WRONG!

TAN IS POSITIVE!!

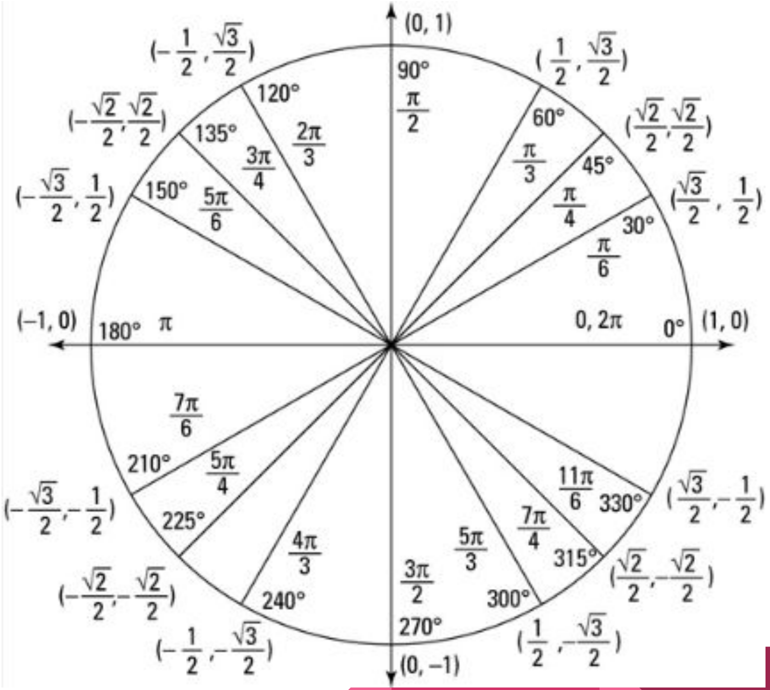
Why? Well, sin and cos are both negative, and tan is sin/cos, so a negative divided by a negative is a positive!

There is a little acronym that will help you remember what's positive and negative.

All **S**tudents **T**ake **C**alculus

All **S**in **T**an **C**os

All is positive in 1, Sin is positive in 2, Tan is positive in 3, and Cos is positive in 4.



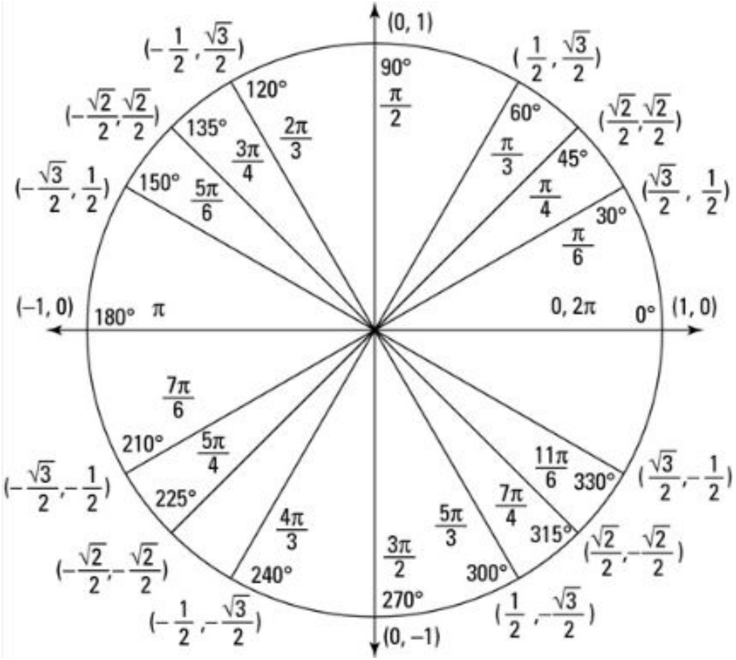
TRIG PRACTICE

FINALLY SOME MATH

Tan(pi/3)

Whatcha think? I'll give you some time.

No rush. No rush at all. Helpful Circle →



Take ur time.

Answer in 3. 2. 1. Nope just kidding. Some time still. Countdown?

NOPE SURPRISE ATTACK ANSWER SHOWING NOWWWW

Tan(pi/3)

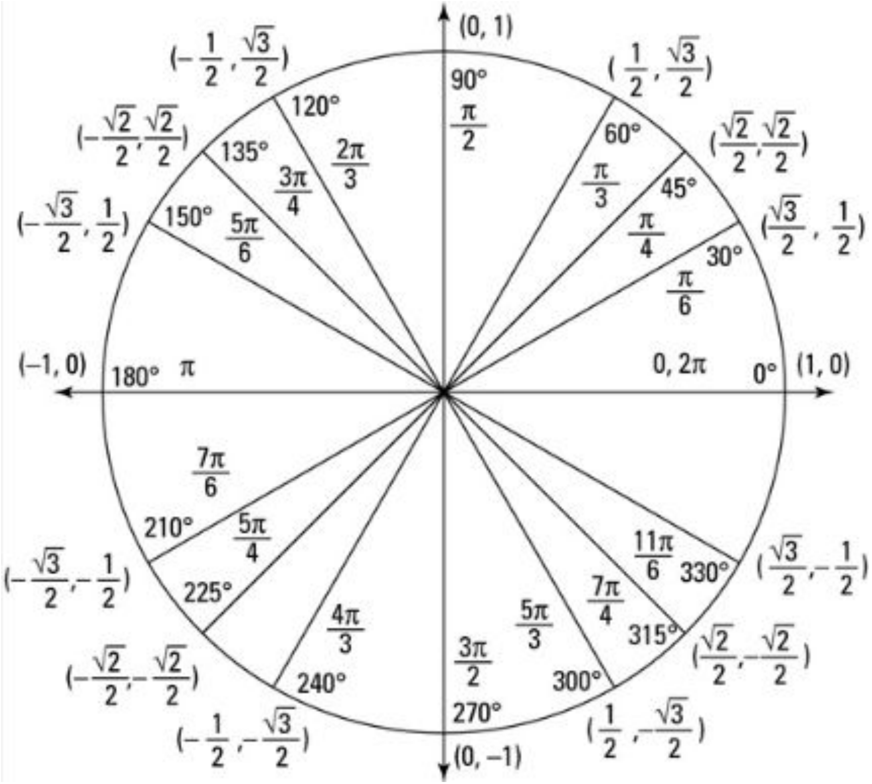
Tan = sin/cos

$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

$\frac{\sin}{\cos}$

$= \frac{\left(\frac{\sqrt{3}}{2}\right)}{2} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$



Tan(390º)

Are ya ready? This a bit tricky right?

I hope so.

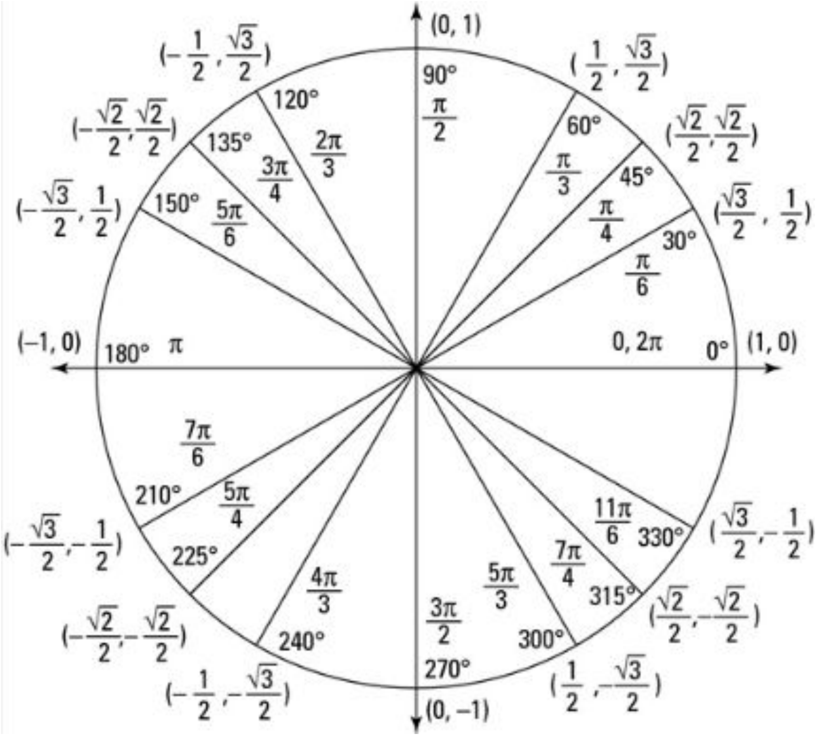
I hope it's difficult.

If it's not difficult I did a bad job :(

You can just pretend to struggle a little bit to make me feel better.

Ik yall too smart for this.

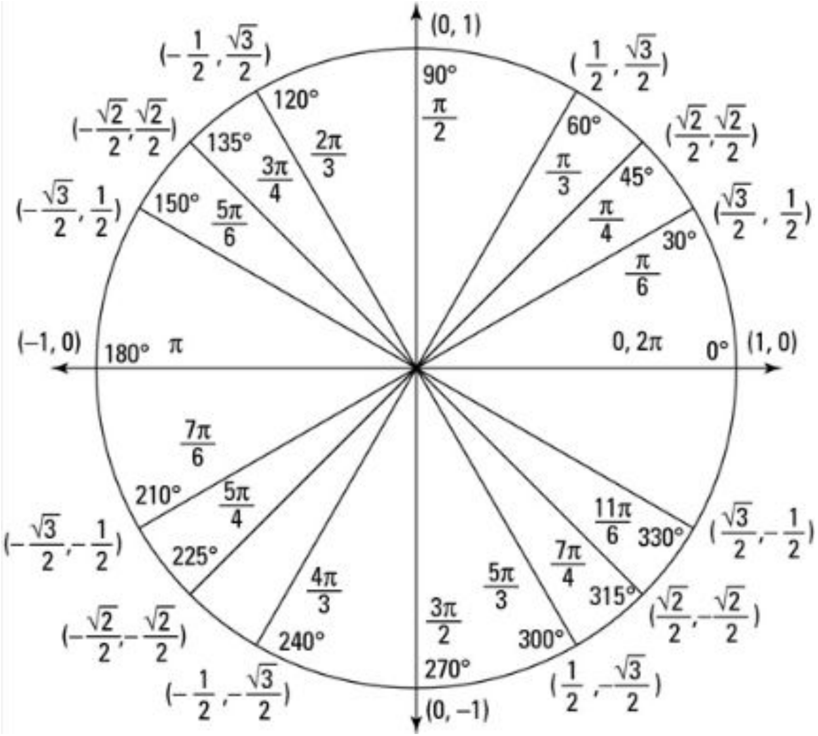
ANSWER REVEALLLLL



Tan(390º)

$$\tan(390) = \tan(30)$$

$$= \frac{\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$



Ok, I get it, you’ve had enough

TIME FOR MORE AHHAHHAHAHHAHAHHHAH

BUT NOT THE NORMAL WAYYYY

BUCKLE YER SEATBELTS’

KAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAHOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOOT