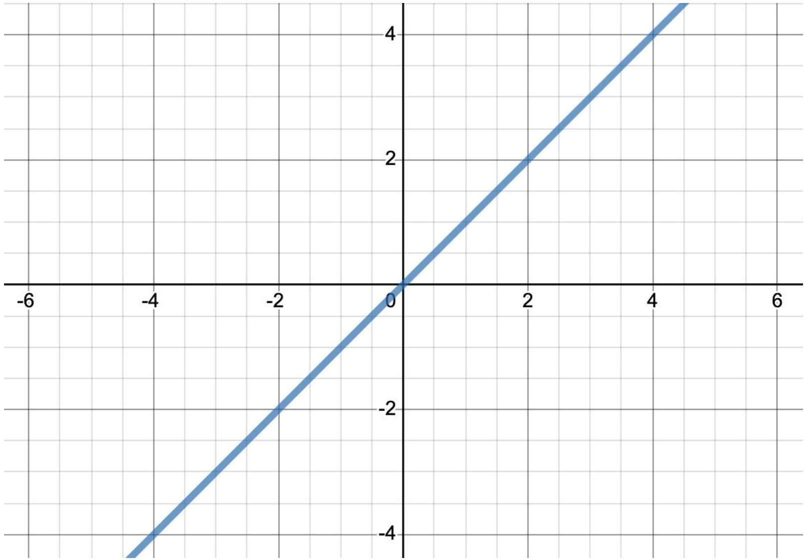
# Precalculus

with Ace

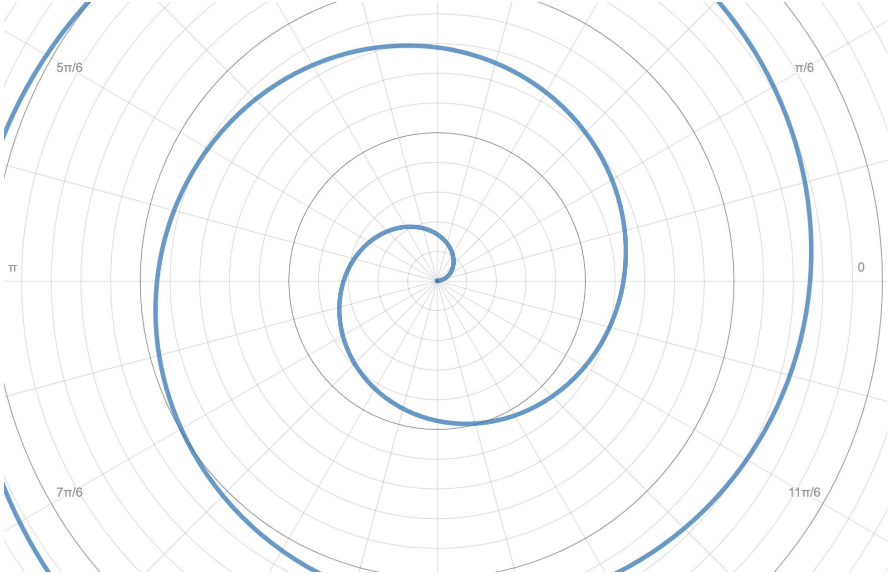
# Functions Can Come in Many Forms…




$y = x$

Rectangular Form

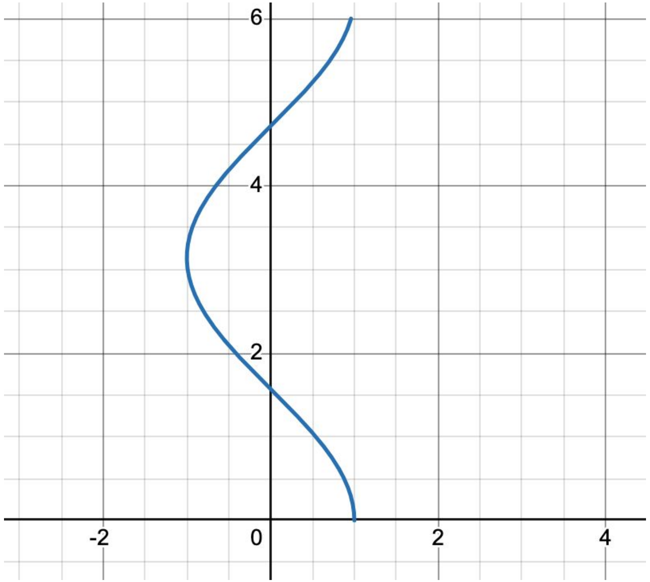
# Functions Can Come in Many Forms...




$r = \theta$

Polar Form

# Functions Can Come in Many Forms...

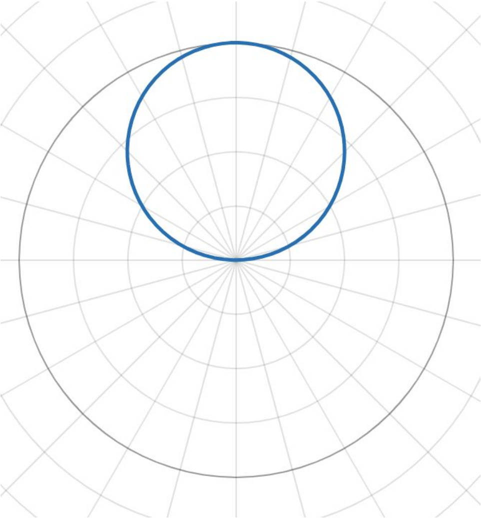



$x(t) = \cos(t)$

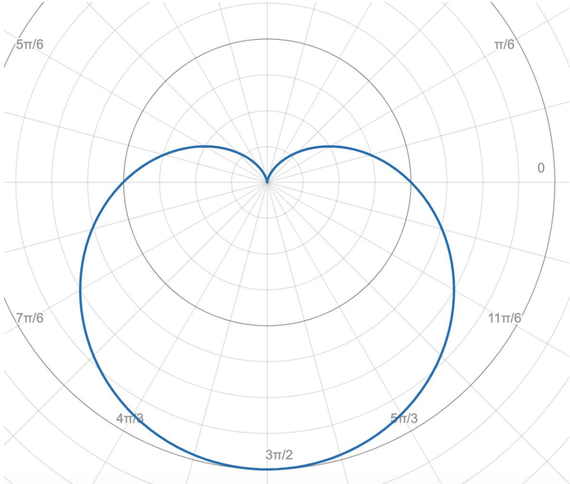
$y(t) = t$

Parametric Form

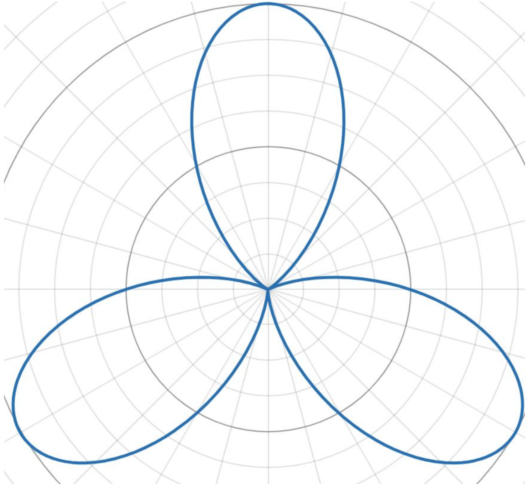
# Let’s Look at Polar Form!



$r = 2\sin(\theta)$

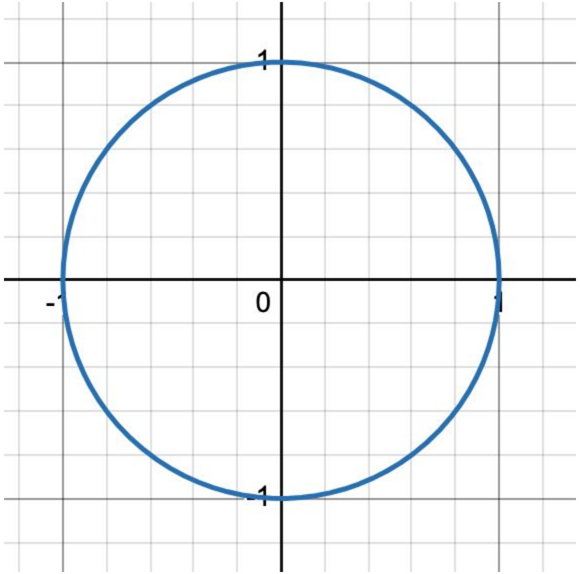



$r = 2 - 2\sin(\theta)$

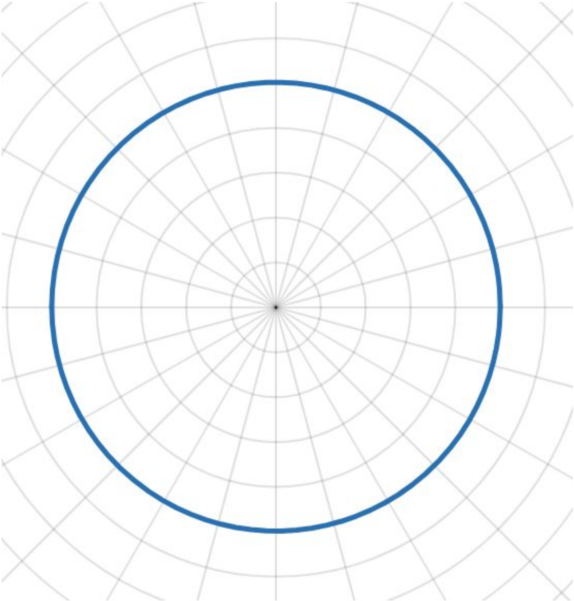


$r = 2 - 2\sin(3\theta)$

# The Same Function Can Be...



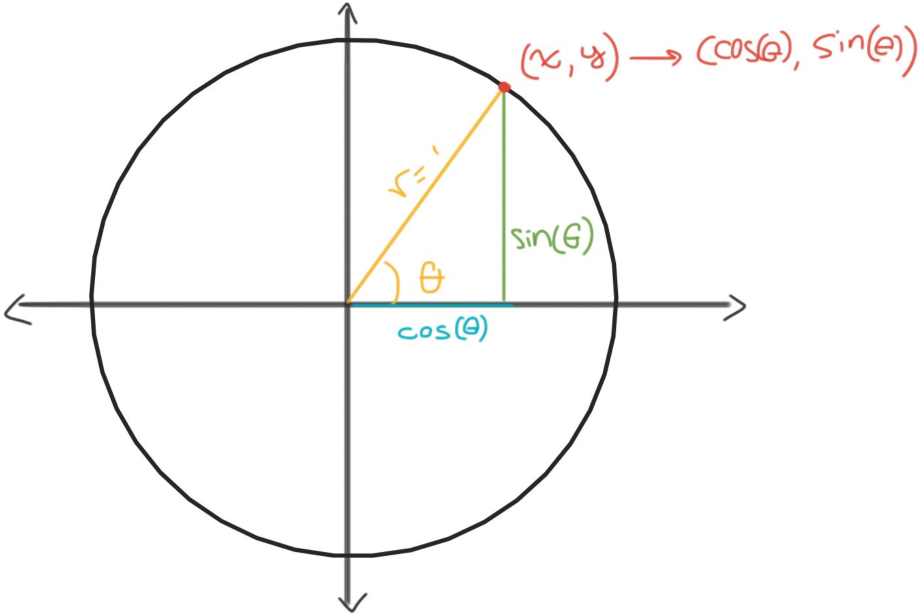

$x^2 + y^2 = 1$



$r = 1$

# Converting to Polar Form

We use trigonometry! Specifically, we use these facts from the unit circle:



# Converting to Polar Form

So, the following holds:

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{y}{x}$$

so $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$x = r\cos(\theta)$

and $y = r\sin(\theta)$

# Converting to Polar Form

So, the following holds:

$$x^2 = r^2\cos^2(\theta)$$
$$y^2 = r^2\sin^2(\theta)$$
$$x^2 + y^2 = r^2(\cos^2(\theta) + \sin^2(\theta))$$

$$\longrightarrow \quad r = \sqrt{x^2 + y^2}$$

# Problem 1

Convert the following coordinate pairs to polar form:

I. (1, 2)
II. (2, 4)
III. (3, 6)

# Problem 2

Convert the following coordinate pairs to rectangular form:

I. $(1, \pi/2)$
II. $(8, 2\pi/3)$
III. $(4, \pi)$

# Problem 3

Convert the following equations to polar form:

I. $2x^2 + 2y^2 = 1$
II. $y = \sqrt{x}$

# Problem 4

Convert the following equations to rectangular form:

I. $r = \sin(2\theta)$

II. $r = 2 - 2\sin(\theta)$

# Let's Talk Rational Functions

These are quotients of polynomials! They may have holes or asymptotes. Can you tell which will have a hole and which an asymptote, and where?

1) $$\frac{x^2+2x-15}{x^2-9}$$

2) $$\frac{x^2+2x-15}{x^2-6x+9}$$

# Let's Talk Rational Functions

How about now?

1) $$\frac{(x+5)(x-3)}{(x-3)(x+3)}$$

2) $$\frac{(x+5)(x-3)}{(x-3)^2}$$

# Let's Talk Rational Functions

Rational functions are also interesting for other reasons—such as **end behavior**. This is how they behave at the ends of their domains. You can determine end behavior (i.e., presence of a horizontal asymptote or not) by looking at the **leading coefficients** of the numerator and denominator. If…

$$\frac{ax^n}{bx^m}$$

1) $n > m$ → no asymptote

2) $n = m$ → $y = \frac{a}{b}$

3) $n < m$ → $y = 0$

# Problem 5

List the end behaviors as $x \to \infty$ for the functions a(x) through c(x).

I.

II.

III.

$$a(x) = \frac{2x^2 + \sqrt{x}}{x\sqrt{x}}$$

$$b(x) = \frac{8x^2 - 8}{(\frac{1}{2}x - 1)(x + 2)}$$

$$c(x) = \frac{x^5 + 3x^4 - x^3 + x^2}{x^4 - 3x^3 + 9x^2 - x^7}$$

# Problem 6

Which of the following statements is true about f(x)?

I. It has no vertical asymptotes.
II. It has no horizontal asymptotes.
III. It has a removable discontinuity at x = -1.

$$f(x) = \frac{x^4 + 10x^3 + 35x^2 + 50x + 24}{x^3 - x^2 - 2x}$$

# Calculus Preview

with Ace

# Derivatives

The reason you can't find the slope of a parabola by taking $\Delta y/\Delta x$ is that the slope is constantly changing! That's why the shape is curved.

So algebra can't help you. But a calculus concept can. Calculus is designed to handle changing slopes. A **derivative** is a function that helps you find the slope of another function at any point. Just like you can plug in a value $a$ into a function $f(x)$ to find the y-value, you can plug in the same value $a$ into the function's derivative, $f'(x)$, to find its slope at $x = a$.

# Derivatives

But how do you find a function’s derivative?

Techniques vary. But here’s a trick for polynomials.

For each term in the polynomial, you take the power and multiply it by the term’s coefficient. Then, you decrease the power by 1. If there’s a constant term, you just leave that out. It’s that simple.

$$2x^2 + 4x - 1 \longrightarrow 4x^{2-1} + 4x^{1-1}$$

$$2x^2 + 4x^1 - \not{1} \qquad 4x + 4$$

# Tangent Line Equations

Once you've found the derivative, you know the slope of a function at that point. You can then use it to write the equation of the **tangent line** to the function at that point.

The tangent line is just a line that (usually) touches a function only at one point. In this case, it would touch the function exactly at the x-value for which you calculated the derivative.

To write the tangent line, you need the slope (which is the derivative at that point) and a point—the point it is supposed to touch.

# Tangent Line Equations

So, to write the equation of a tangent line:

1. Find the derivative of the function.
2. Find the slope of the function at that point.
3. Use the coordinates of the point, and the derivative, to write the equation. (Use point-slope form.)

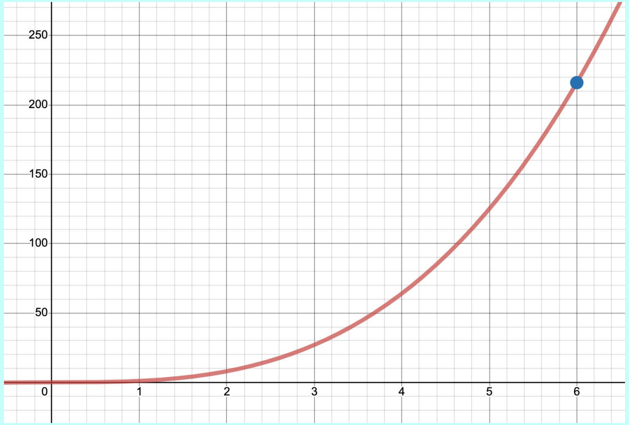
# Normal Line Equations

The **normal line** is just like the tangent line—except, instead of being tangent to a function at a point, the normal line is perpendicular to the function at that point.

Since it's perpendicular to the function's tangent line as well, its slope is the negative reciprocal of the tangent line. You can use this fact to find the normal line given a tangent line.

# Problem 7

Find the equation of the tangent line at $x = 6$ for $f(x) = x^3$.



# Problem 8

The equation of the tangent line to $f(x) = \sqrt{x}$ at $x = 4$ is

$$y = x/4 + 1$$

Identify $f'(4)$ and then write the equation of the normal line to $f(x)$ at $x = 4$.

# Integrals

Calculus is also good for finding areas!

It's easy to find areas under lines (just use geometric formulas), but not so easy to find areas under parabolas or other curved functions, at least with algebra. With calculus, you can actually find these areas and more.

That's what an **integral** is used for.

For our purposes today, though, we'll focus on a special property of integrals—the process of finding an integral is the opposite of finding a derivative.

# Integrals

To find an integral of a function, you do the reverse of finding the derivative. For many functions, this is not very straightforward; however, for polynomials, it is. You just undo the steps you usually do when finding the derivative. That is, add one to the exponent of each term and divide the terms' coefficients by those new exponents.

Also, remember to add a constant, C, at the end. This represents any constant you might have lost when taking a derivative.

# Integrals

Example:

$$\int (2x^2 + 4x - 1)\,dx$$

$$= \frac{2}{3}x^{2+1} + \frac{4}{2}x^{1+1} - \frac{1}{1}x^{0+1} + C$$

$$= \frac{2}{3}x^3 + 2x^2 - x + C$$

# Problem 9

Find the indefinite integral of the following functions:

I. $f(x) = 2x^2$
II. $g(x) = \sqrt{x} + 1/x^2$

# Integrals

To find the area under a function over an interval (say, from x = a to x = b), you just take the antiderivative ($F(x)$, which is what you've been finding so far) and do the following:

$$F(\text{b}) - F(\text{a})$$

So, if the interval is [1, 3], you would do the reverse process of finding the derivative on the function $f(x)$ to get $F(x)$, and find $F(3) - F(1)$.

# Problem 10

Find the integral of the following functions on the given intervals:

I. $f(x) = 2x^2$ on $[-1, 1]$
II. $g(x) = \sqrt{x} + 1/x^2$ on $[2, 3]$

# The End of Mathathon!

**Congratulations! You made it! Great job :)**